Linear Perspective

CS418 Computer Graphics

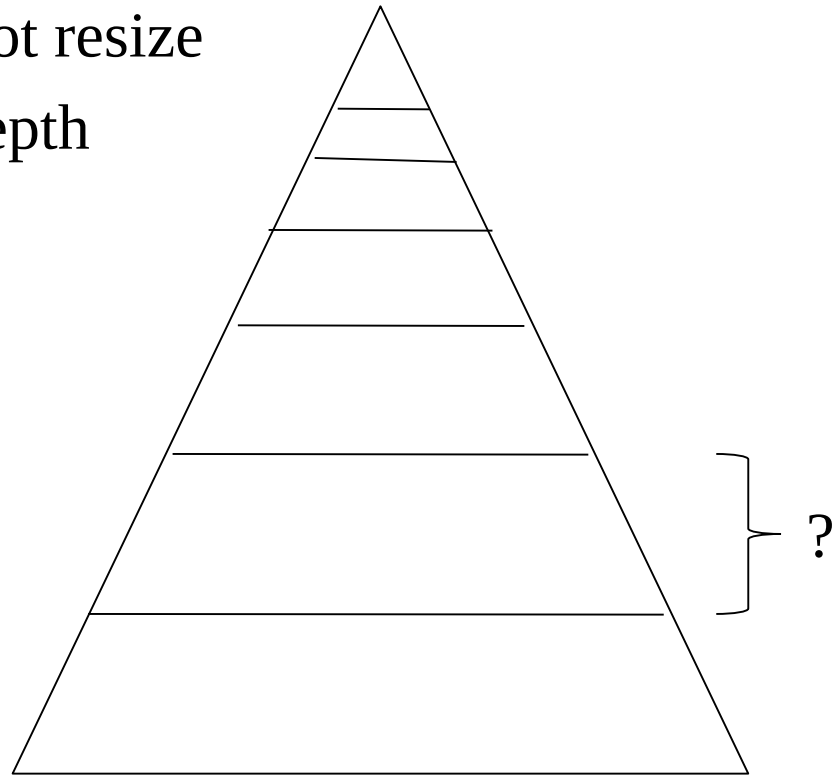
John C. Hart

Linear Perspective

- Brain depends on shape constancy
  - Real world objects do not resize
  - Change in size due to depth
- Closer objects larger
- Farther objects smaller
- How large, how small?

Linear Perspective

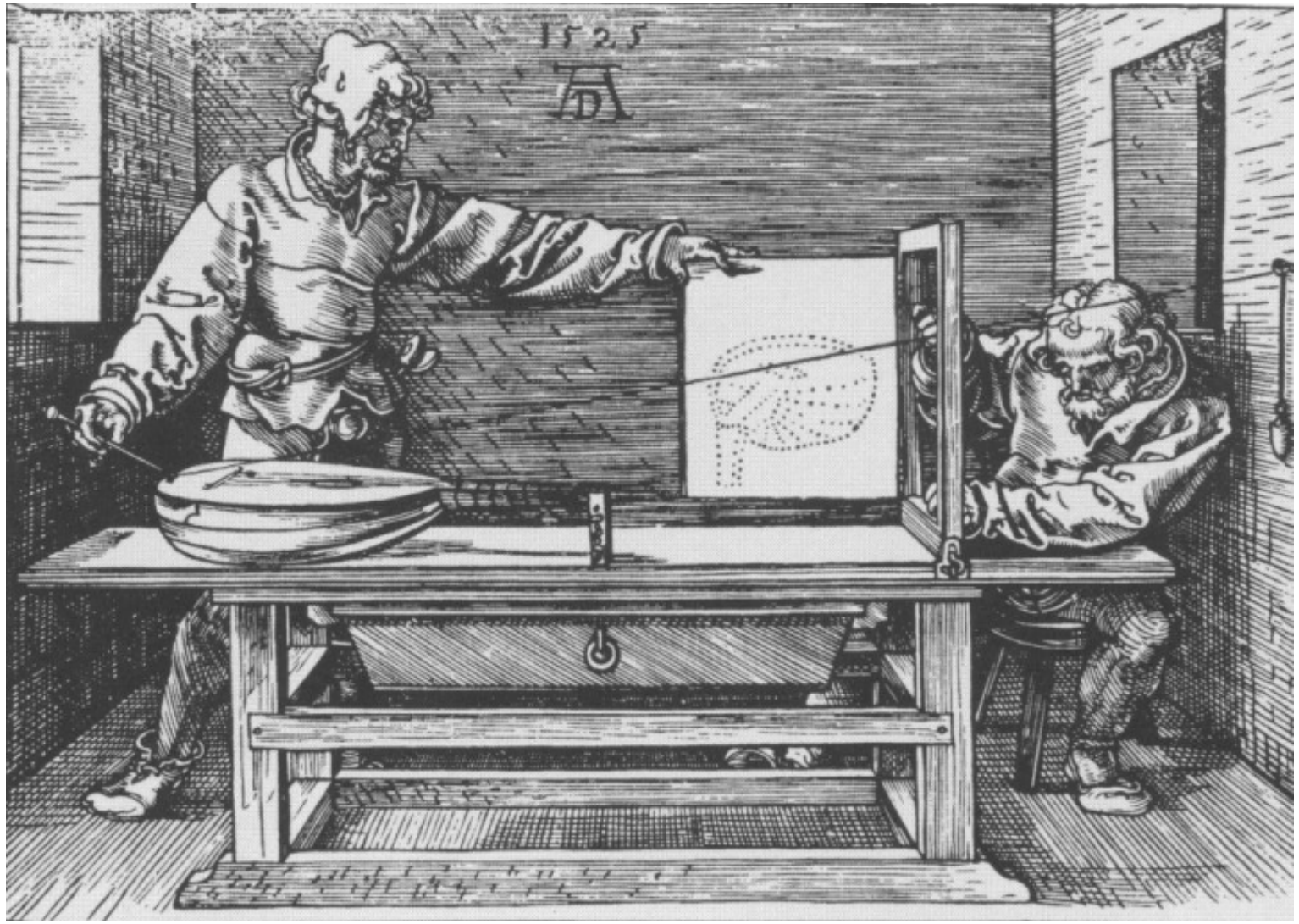
- Brain depends on shape constancy
  - Real world objects do not resize
  - Change in size due to depth
- Closer objects larger
- Farther objects smaller
- How large, how small?



Linear Perspective

Albrecht Durer woodcut c. 1525,
swiped from Marc Levoy's CS48N notes c. 2007

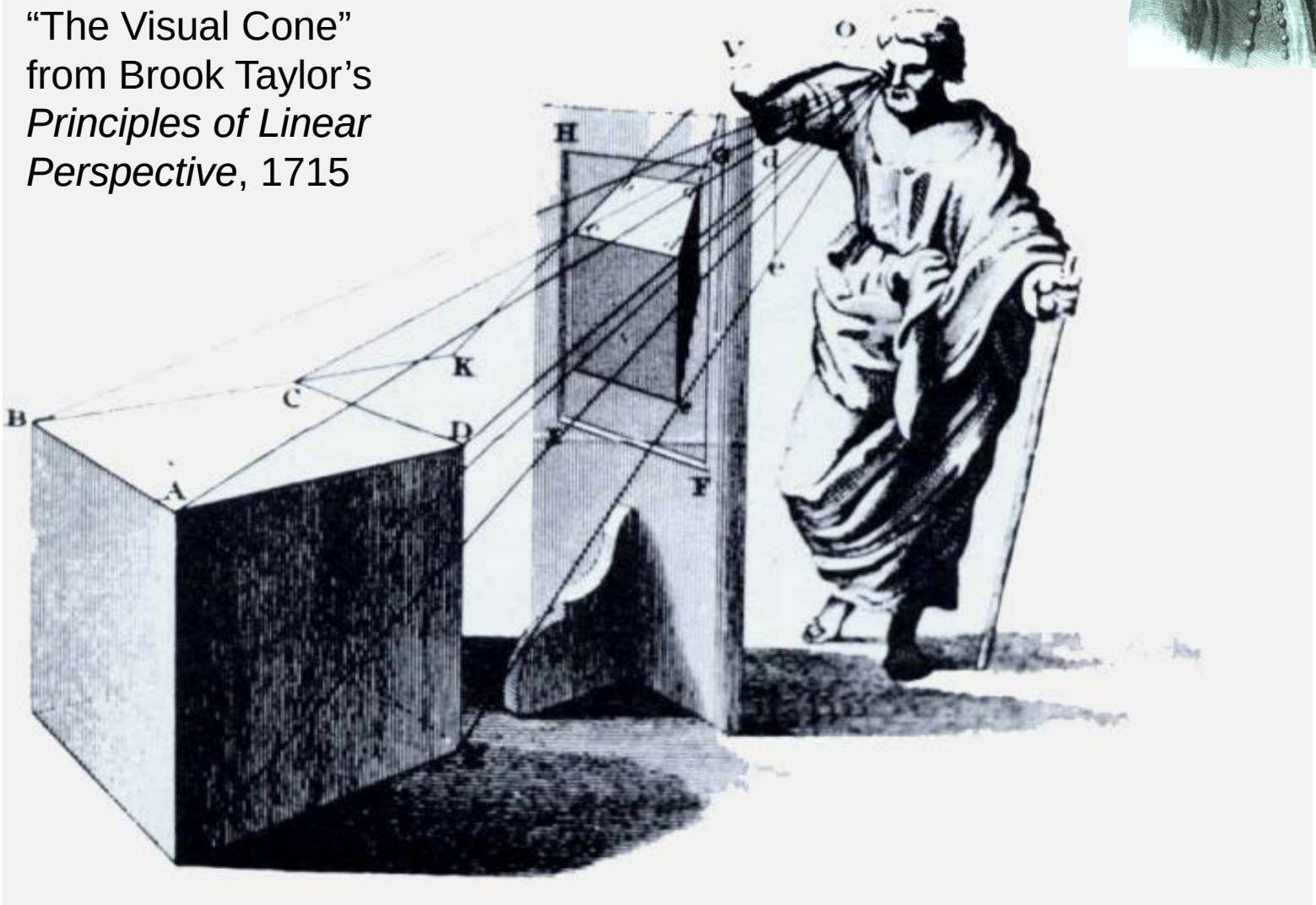
Linear Perspective



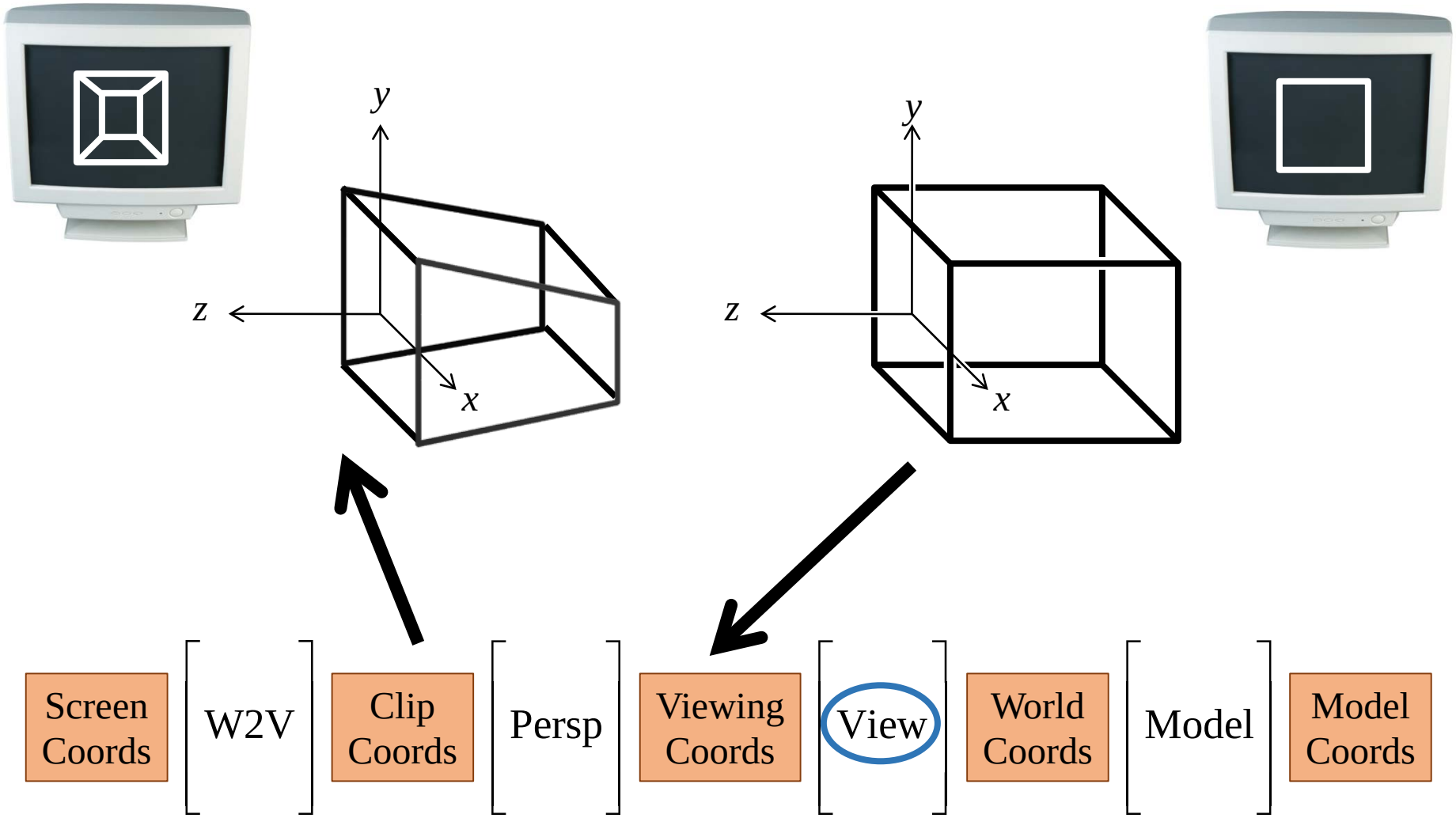
More Durer, swiped from Fredo Durand's Art of Depiction

Linear Perspective

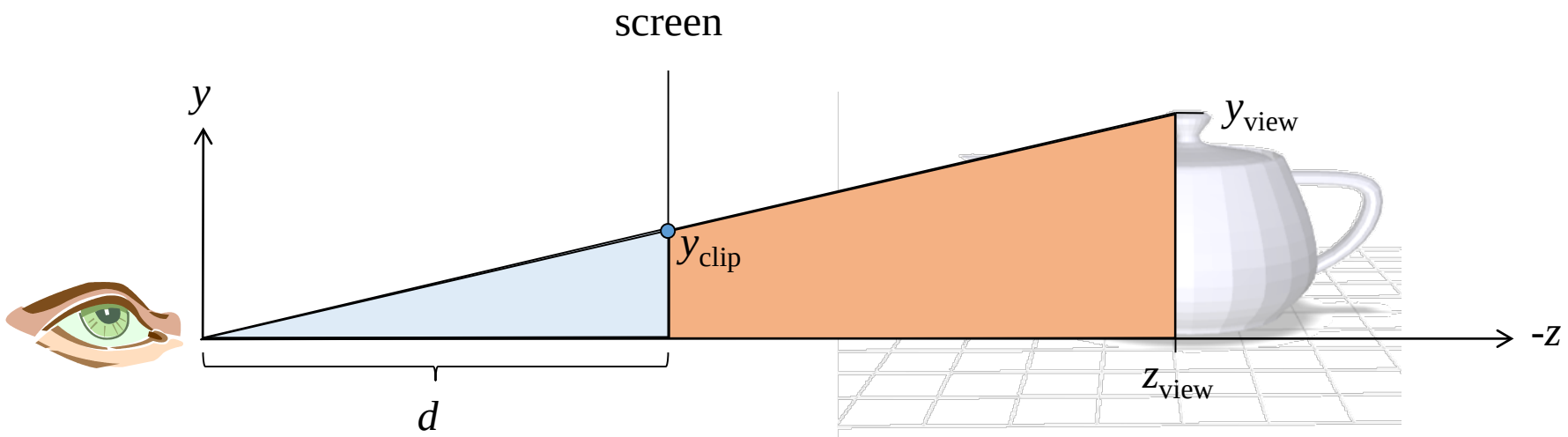
“The Visual Cone”
from Brook Taylor’s
Principles of Linear Perspective, 1715



Vertex Pipeline



Linear Perspective



$$\frac{y_{clip}}{d} = \frac{y_{view}}{-z_{view}}$$

$$y_{clip} = d\,\frac{y_{view}}{-z_{view}} = \frac{y_{view}}{-z_{view}/d}$$

Homogeneous Coordinates

- Fourth homogeneous coordinate can be any non-zero value

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \equiv \begin{bmatrix} wx \\ wy \\ wz \\ w \end{bmatrix}$$

Homogeneous Coordinates

- Fourth homogeneous coordinate can be any non-zero value
- To find the point it corresponds to:
  - multiply all four coordinates by the same value
  - precisely the value that makes the fourth coordinate one

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \equiv \begin{bmatrix} wx \\ wy \\ wz \\ w \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \equiv \begin{bmatrix} x/w \\ y/w \\ z/w \\ 1 \end{bmatrix}$$

Homogeneous Coordinates

- Fourth homogeneous coordinate can be any non-zero value
- To find the point it corresponds to:
  - multiply all four coordinates by the same value
  - precisely the value that makes the fourth coordinate one

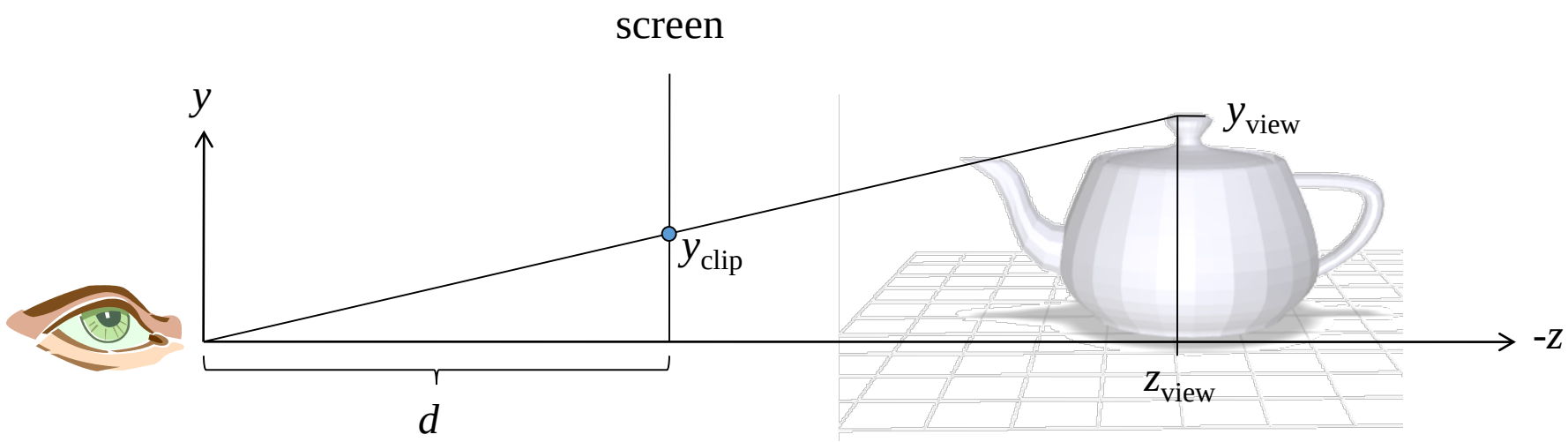
$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \equiv \begin{bmatrix} wx \\ wy \\ wz \\ w \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \equiv \begin{bmatrix} x/w \\ y/w \\ z/w \\ 1 \end{bmatrix}$$

- When homogeneous coordinate is zero
  - Denotes a “point” at infinity
  - Represents a vector instead of a point
  - Not affected by translation

$$\begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & & & a \\ & 1 & & b \\ & & 1 & c \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$$

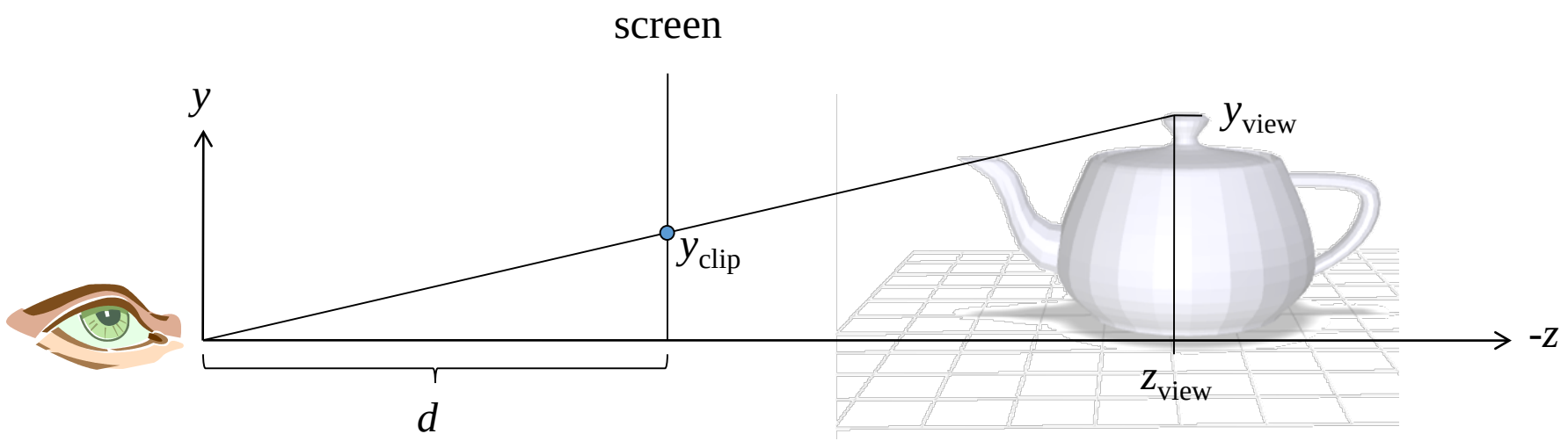
Linear Perspective



$$\frac{y_{clip}}{d} = \frac{y_{view}}{-z_{view}}$$

$$y_{clip} = \frac{y_{view}}{-z_{view} / d}$$

Linear Perspective

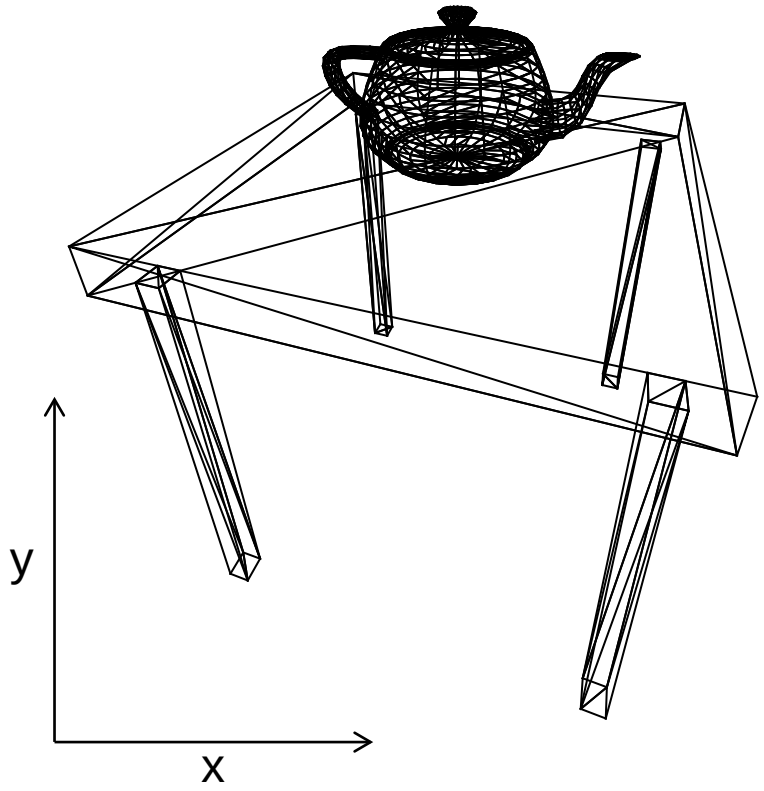


$$\frac{y_{clip}}{d} = \frac{y_{view}}{-z_{view}}$$

$$y_{clip} = \frac{y_{view}}{-z_{view}/d}$$

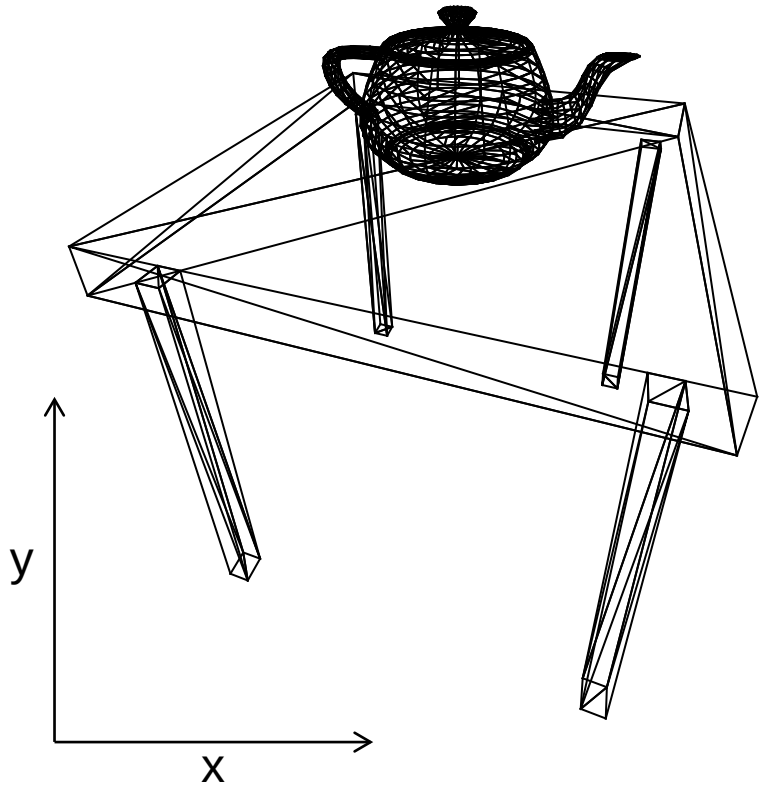
$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & -1/d & 0 \end{bmatrix} \begin{bmatrix} x_{view} \\ y_{view} \\ z_{view} \\ 1 \end{bmatrix} = \begin{bmatrix} x_{view} \\ y_{view} \\ z_{view} \\ -z_{view}/d \end{bmatrix} \equiv \begin{bmatrix} \frac{x_{view}}{-z_{view}/d} \\ \frac{y_{view}}{-z_{view}/d} \\ -d \\ 1 \end{bmatrix}$$

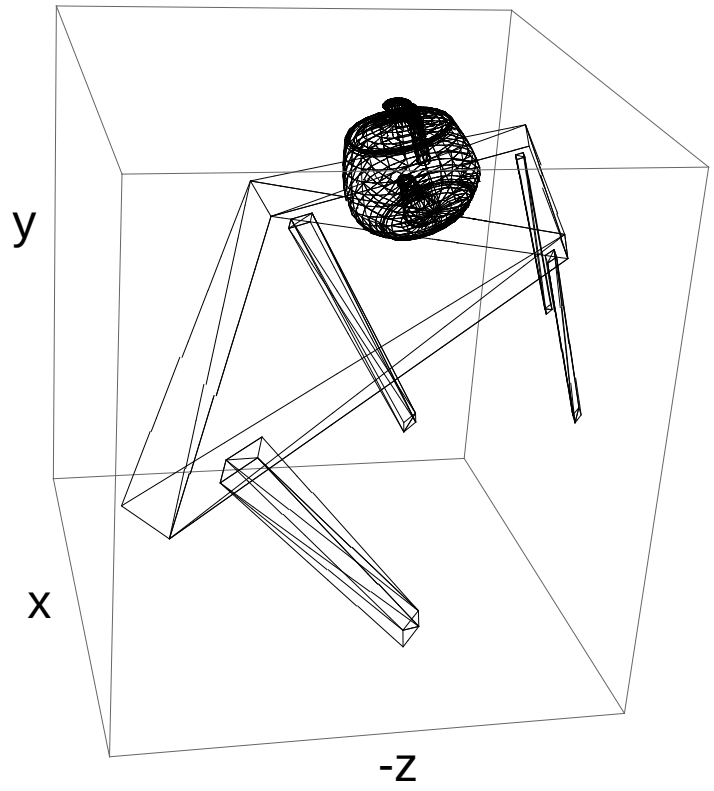
Perspective Distortion



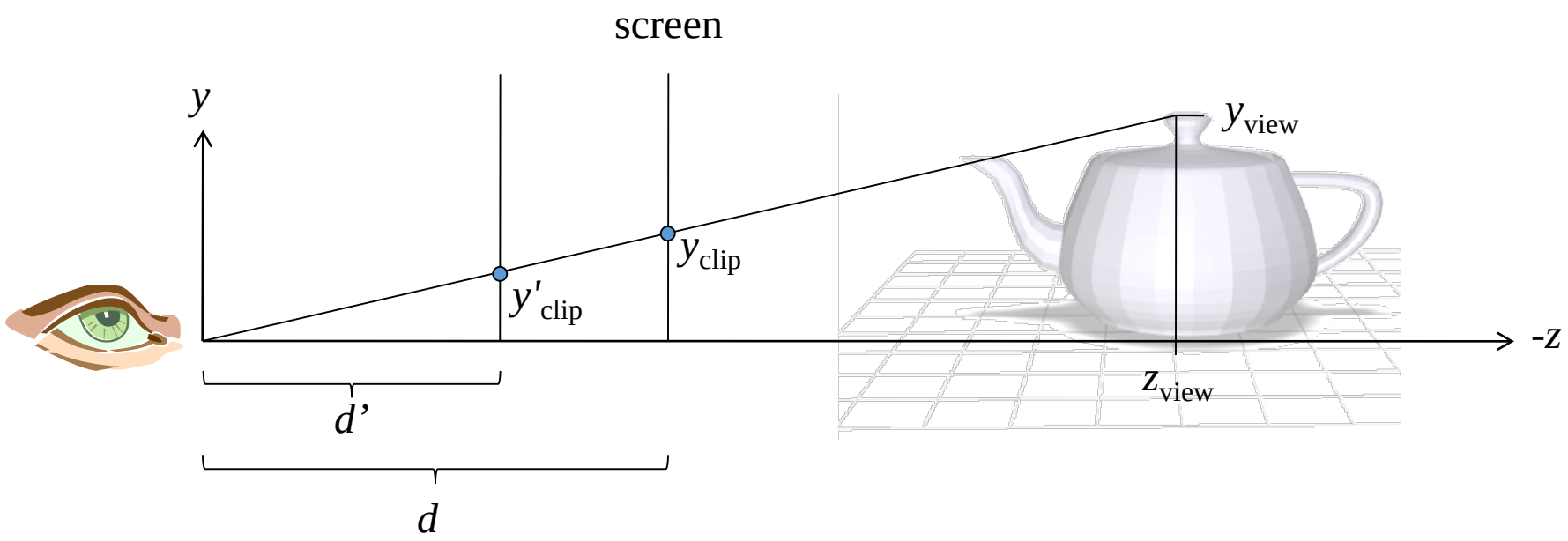
Perspective Distortion

(Using a later version of Perspective matrix that preserves depth ordering)





Parameter d

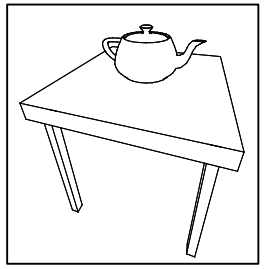


$y_{clip} = d\ y_{view}/(-z_{view})$

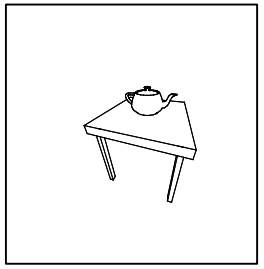
$y'_{clip} = d'\ y_{view}/(-z_{view}) = (d'/d)\ y_{clip}$

Changing parameter d just changes scale of projection

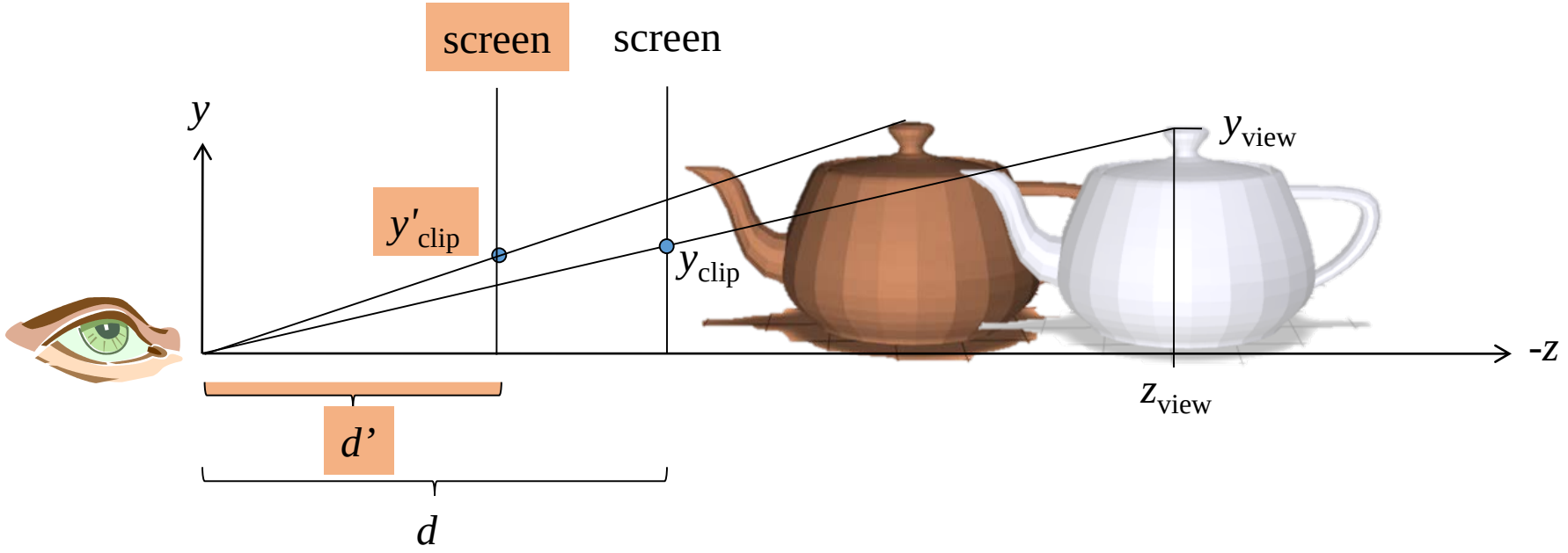
Using d



Using d'



Parameter d

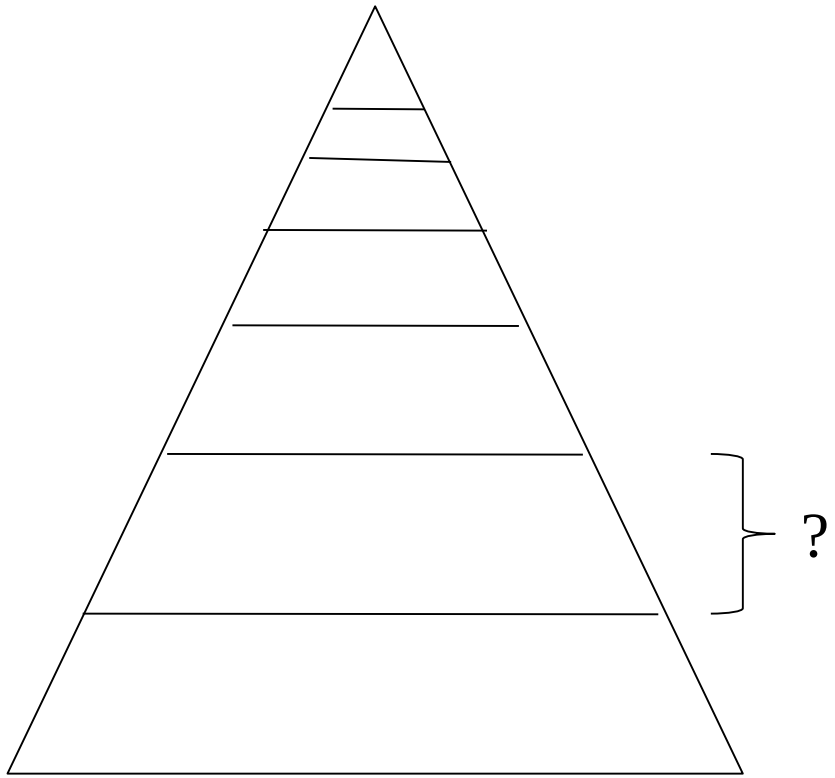


To change degree of perpective distortion, need to change distance from eye to scene,

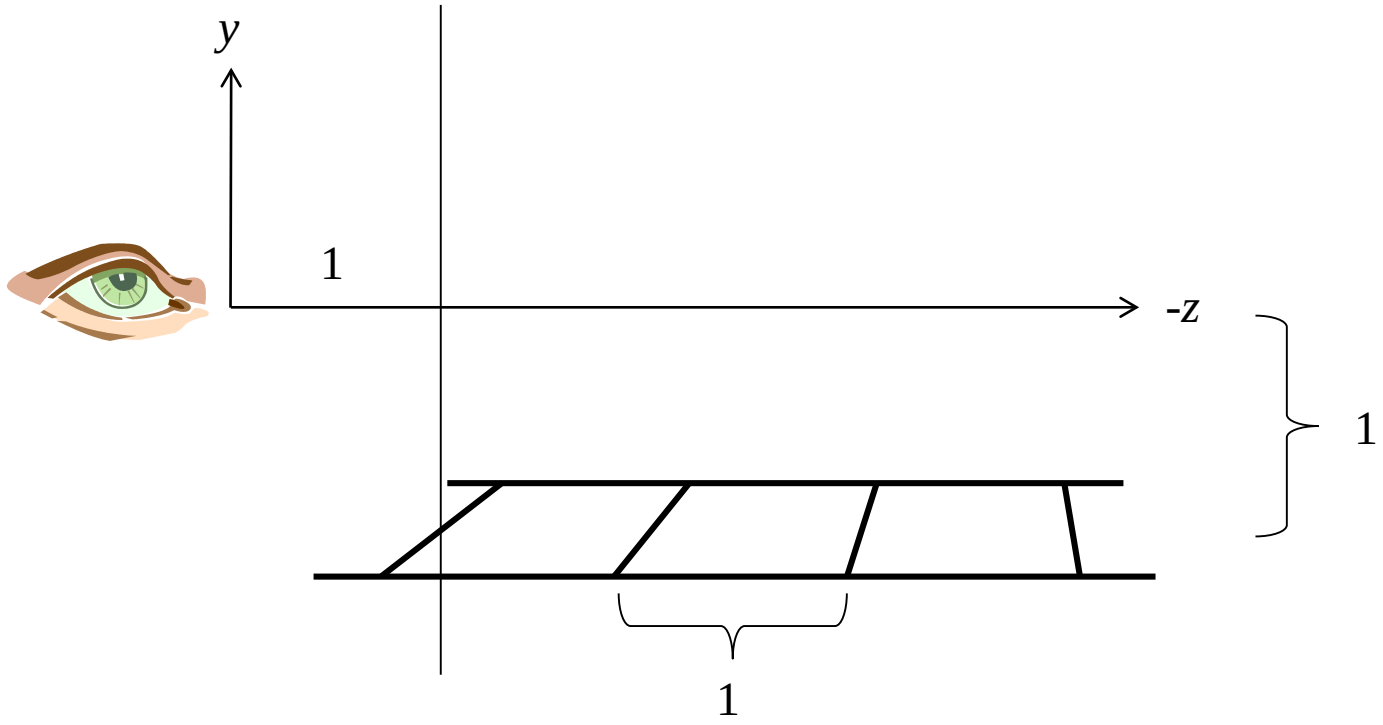
…by moving scene closer or farther to eye,

… along z axis in viewing coordinates

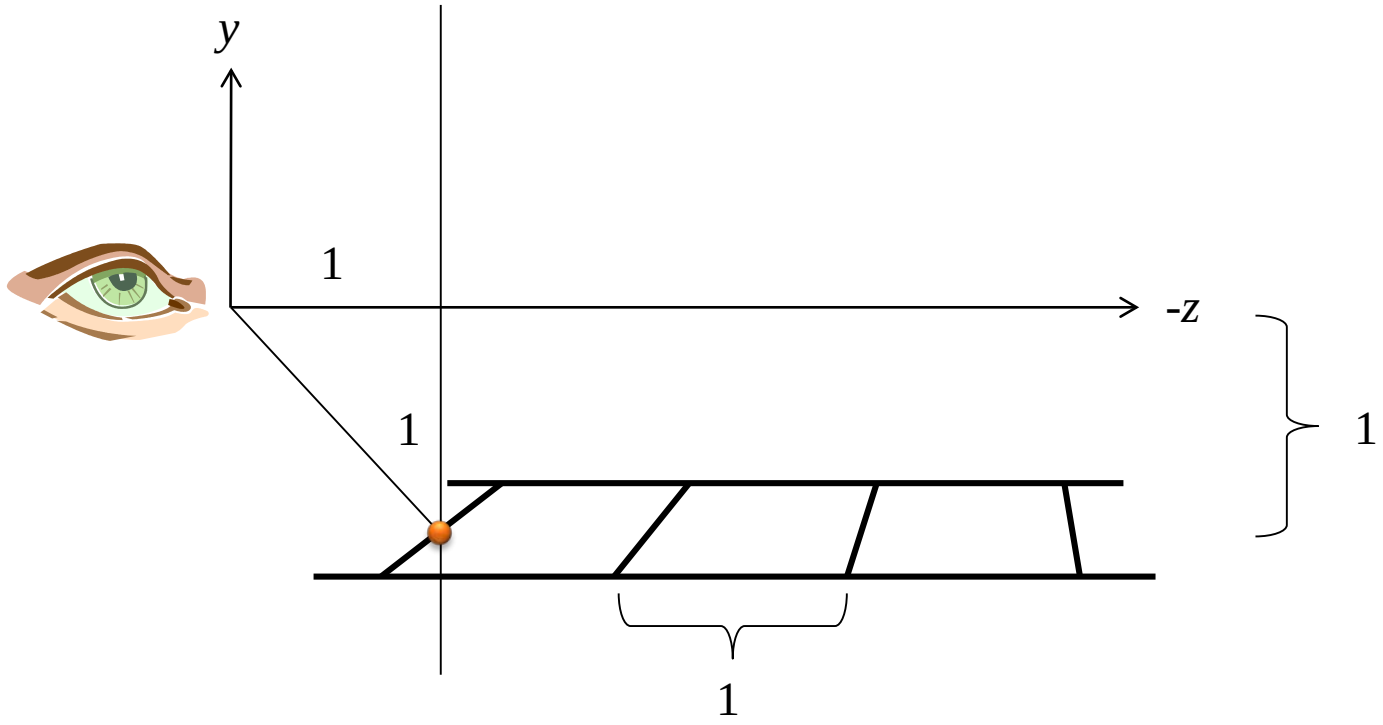
Sidewalks



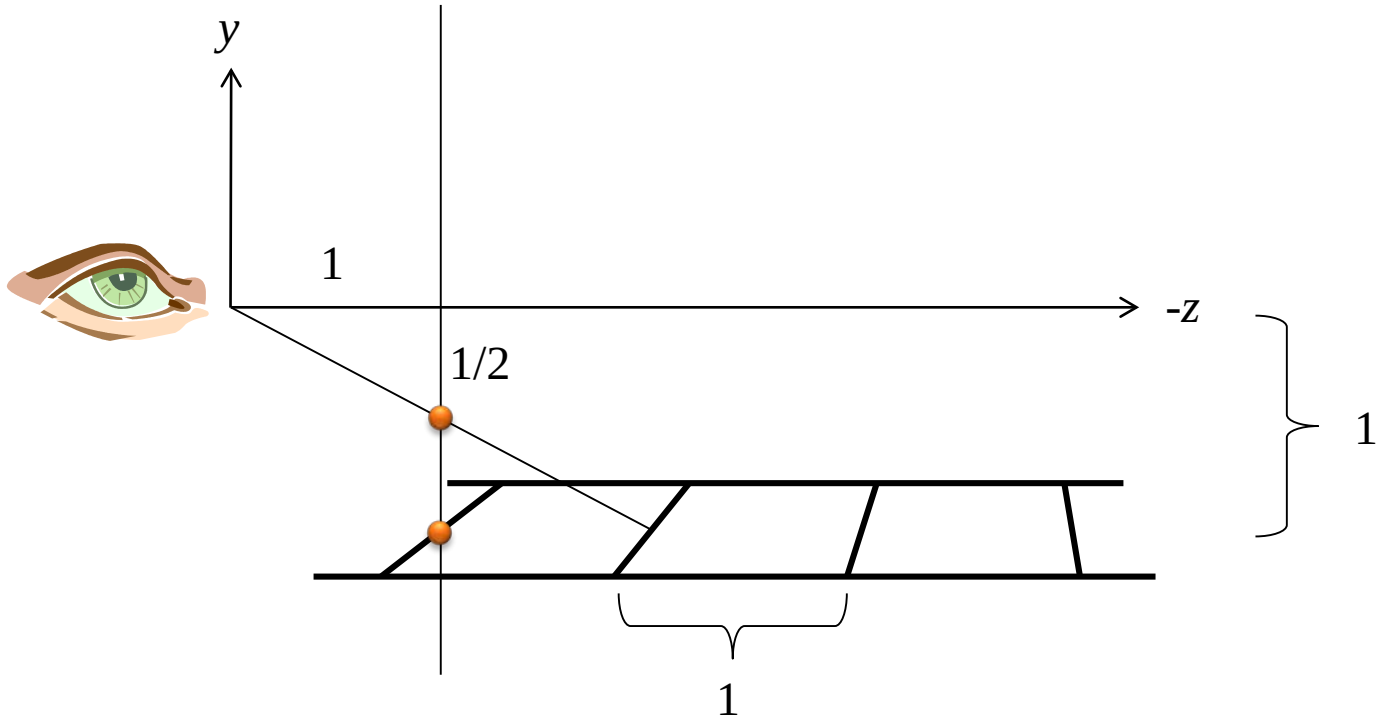
Sidewalks



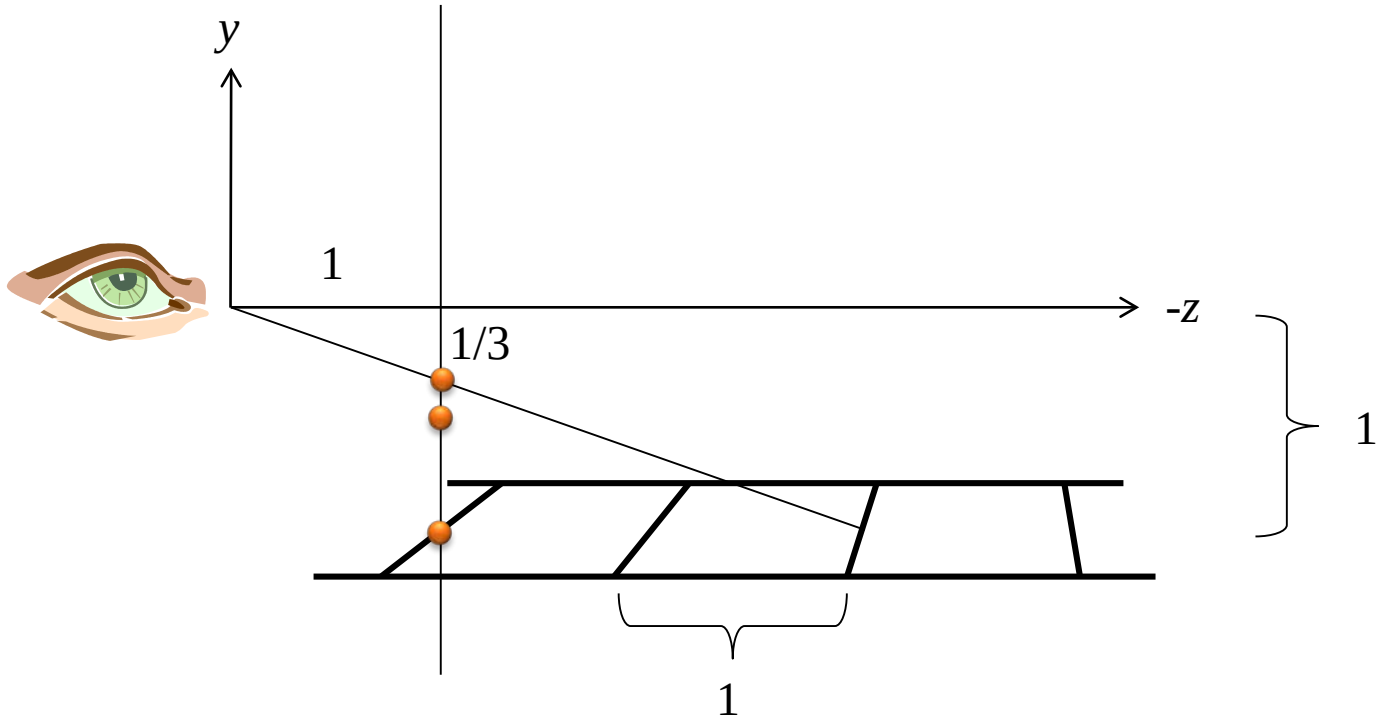
Sidewalks



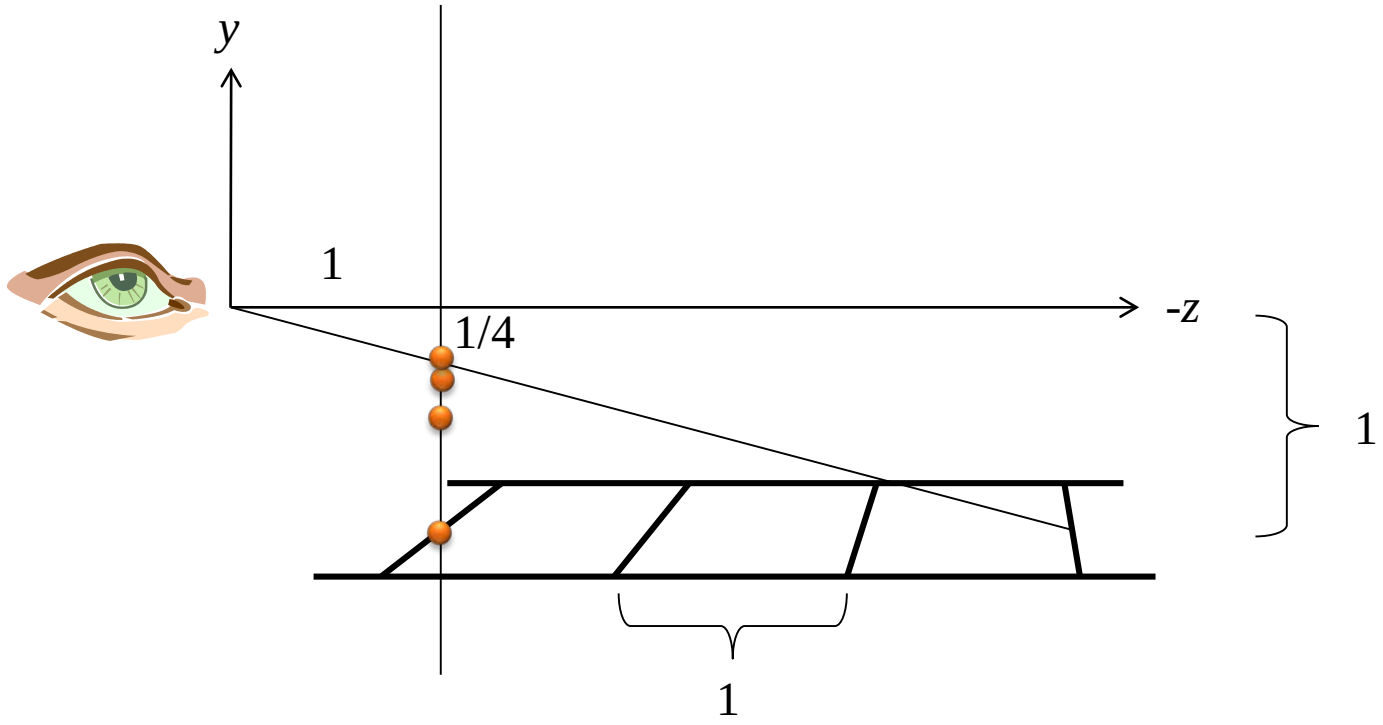
Sidewalks



Sidewalks



Sidewalks



Sidewalks